

# Nonproportional Relationships

MODULE

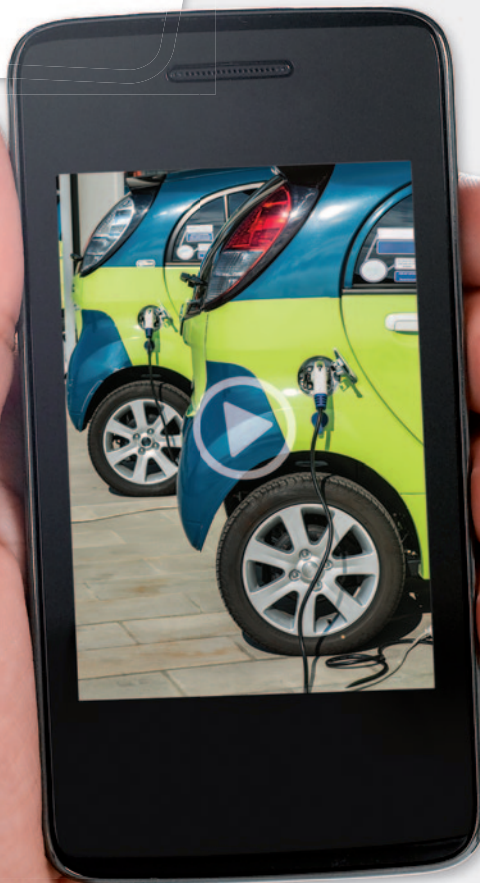


# 4



## ESSENTIAL QUESTION

How can you use non-proportional relationships to solve real-world problems?



LESSON 4.1

### Representing Linear Nonproportional Relationships

FL 8.F.1.3

LESSON 4.2

### Determining Slope and $y$ -intercept

FL 8.EE.2.6, 8.F.2.4

LESSON 4.3

### Graphing Linear Nonproportional Relationships using Slope and $y$ -intercept

FL 8.F.1.3, 8.F.2.4

LESSON 4.4

### Proportional and Nonproportional Situations

FL 8.F.1.2, 8.F.1.3, 8.F.2.4



### Real-World Video

The distance a car can travel on a tank of gas or a full battery charge in an electric car depends on factors such as fuel capacity and the car's efficiency. This is described by a nonproportional relationship.

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# Are YOU Ready?

Complete these exercises to review skills you will need for this module.



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## Integer Operations

**EXAMPLE**  $-7 - (-4) = -7 + 4$   
 $|-7| - |4|$   
 $7 - 4, \text{ or } 3$   
 $= -3$

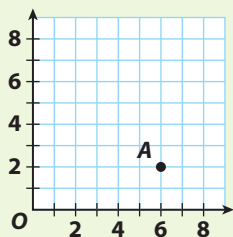
To subtract an integer, add its opposite.  
The signs are different, so find the difference of the absolute values.  
Use the sign of the number with the greater absolute value.

Find each difference.

- $3 - (-5)$  \_\_\_\_\_
- $-4 - 5$  \_\_\_\_\_
- $6 - 10$  \_\_\_\_\_
- $-5 - (-3)$  \_\_\_\_\_
- $8 - (-8)$  \_\_\_\_\_
- $9 - 5$  \_\_\_\_\_
- $-3 - 9$  \_\_\_\_\_
- $0 - (-6)$  \_\_\_\_\_
- $12 - (-9)$  \_\_\_\_\_
- $-6 - (-4)$  \_\_\_\_\_
- $-7 - 10$  \_\_\_\_\_
- $5 - 14$  \_\_\_\_\_

## Graph Ordered Pairs (First Quadrant)

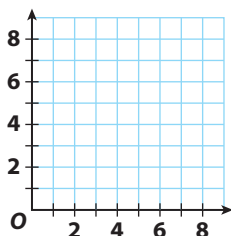
**EXAMPLE**



To graph a point at  $(6, 2)$ , start at the origin.  
Move **6 units right**.  
Then move **2 units up**.  
Graph point  $A(6, 2)$ .

Graph each point on the coordinate grid.

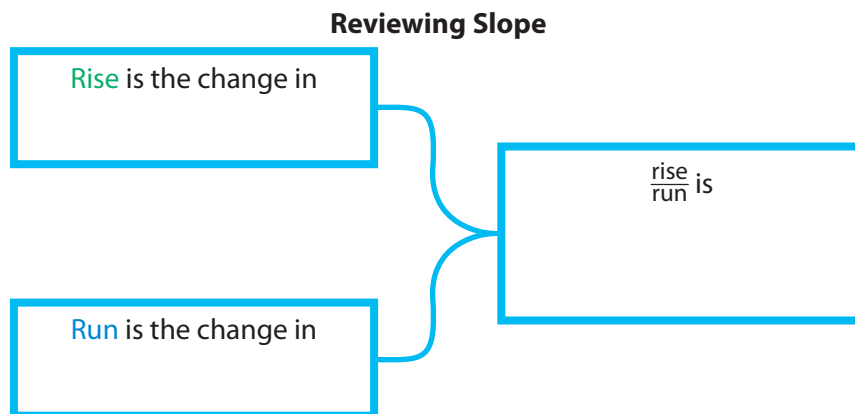
- $B(0, 5)$
- $C(8, 0)$
- $D(5, 7)$
- $E(2, 3)$



# Reading Start-Up

## Visualize Vocabulary

Use the ✓ words to complete the diagram. You can put more than one word in each box.



## Understand Vocabulary

Complete the sentences using the preview words.

1. The y-coordinate of the point where a graph of a line crosses the y-axis is the \_\_\_\_\_.
2. A \_\_\_\_\_ is an equation whose solutions form a straight line on a coordinate plane.
3. A linear equation written in the form  $y = mx + b$  is the \_\_\_\_\_.

## Vocabulary

### Review Words

- ordered pair (*par ordenado*)
- proportional relationship (*relación proporcional*)
- ✓ rate of change (*tasa de cambio*)
- ✓ slope (*pendiente*)
- ✓ x-coordinate (*coordenada x*)
- ✓ y-coordinate (*coordenada y*)

### Preview Words

- linear equation (*ecuación lineal*)
- slope-intercept form of an equation (*forma de pendiente-intersección*)
- y-intercept (*intersección con el eje y*)

## Active Reading

**Booklet** Before beginning the module, create a booklet to help you learn the concepts. Write the main idea of each lesson on each page of the booklet. As you study each lesson, write important details that support the main idea, such as vocabulary and formulas. Refer to your finished booklet as you work on assignments and study for tests.





# Unpacking the Standards

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this module.

**FL 8.F.1.3**

Interpret the equation  $y = mx + b$  as defining a linear function whose graph is a straight line.

**Key Vocabulary****slope** (*pendiente*)

A measure of the steepness of a line on a graph; the rise divided by the run.

**y-intercept** (*intersección con el eje y*)

The y-coordinate of the point where the graph of a line crosses the y-axis.

## What It Means to You

You will identify the slope and the y-intercept of a line by looking at its equation and use them to graph the line.

**UNPACKING EXAMPLE 8.F.1.3**

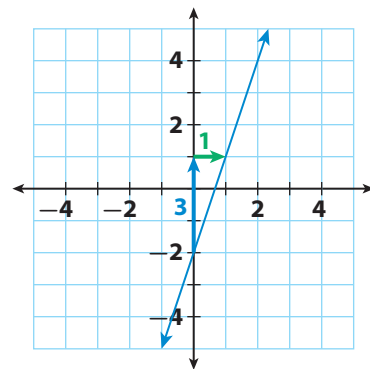
Graph  $y = 3x - 2$  using the slope and the y-intercept.

$$y = mx + b$$

↑                    ↑  
slope            y-intercept

The slope  $m$  is 3, and the y-intercept is  $-2$ .

Plot the point  $(0, -2)$ . Use the slope  $3 = \frac{3}{1}$  to find another point by moving **up 3** and to the **right 1**. Draw the line through the points.

**FL 8.F.1.3**

Give examples of functions that are not linear.

**Key Vocabulary****function** (*función*)

An input-output relationship that has exactly one output for each input.

**linear function** (*función lineal*)

A function whose graph is a straight line.

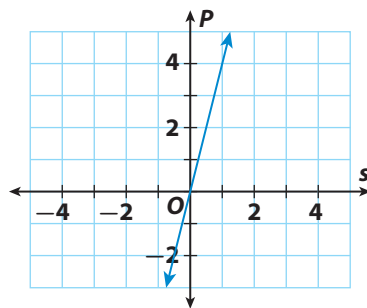
## What It Means to You

You will distinguish linear relationships from nonlinear relationships by looking at graphs.

**UNPACKING EXAMPLE 8.F.1.3**

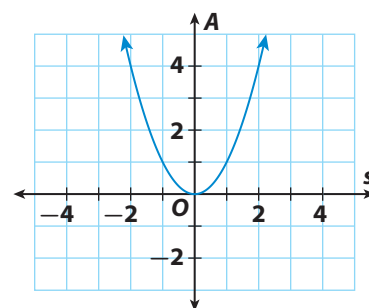
Which relationship is linear and which is nonlinear?

$$P = 4s$$



$P = 4s$  is linear because its graph is a line.

$$A = s^2$$



$A = s^2$  is not linear because its graph is not a line.



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# Representing Linear Nonproportional Relationships



FL 8.F.1.3

Interpret the equation  $y = mx + b$  as defining a linear function, whose graph is a straight line; ...



## ESSENTIAL QUESTION

How can you use tables, graphs, and equations to represent linear nonproportional situations?

## Representing Linear Relationships Using Tables

You can use an equation to describe the relationship between two quantities in a real-world situation. You can use a table to show some values that make the equation true.



### EXAMPLE 1



FL Prep for 8.F.1.3

The equation  $y = 3x + 2$  gives the total charge,  $y$ , for one person to rent a pair of shoes and bowl  $x$  games at Baxter Bowling Lanes based on the prices shown. Make a table of values for this situation.

**STEP 1** Choose several values for  $x$  that make sense in context.

$x$ (number of games)	1	2	3	4
$y$ (total cost in dollars)				

**STEP 2** Use the equation  $y = 3x + 2$  to find  $y$  for each value of  $x$ .

$x$ (number of games)	1	2	3	4
$y$ (total cost in dollars)	5	8	11	14

Substitute 1 for  $x$ :  
 $y = 3(1) + 2 = 5.$



### YOUR TURN

- Francisco makes \$12 per hour doing part-time work on Saturdays. He spends \$4 on transportation to and from work. The equation  $y = 12x - 4$  gives his earnings  $y$ , after transportation costs, for working  $x$  hours. Make a table of values for this situation.

$x$ (number of hours)				
$y$ (earnings in dollars)				



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## Examining Linear Relationships

Recall that a proportional relationship is a relationship between two quantities in which the ratio of one quantity to the other quantity is constant. The graph of a proportional relationship is a line through the origin. Relationships can have a constant rate of change but not be proportional.

The entrance fee for Mountain World theme park is \$20. Visitors purchase additional \$2 tickets for rides, games, and food. The equation  $y = 2x + 20$  gives the total cost,  $y$ , to visit the park, including purchasing  $x$  tickets.

**STEP 1** Complete the table.

$x$ (number of tickets)	0	2	4	6	8
$y$ (total cost in dollars)	20				

**STEP 2** Plot the ordered pairs from the table. Describe the shape of the graph.

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**STEP 3** Find the rate of change between each point and the next. Is the rate constant?

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**STEP 4** Calculate  $\frac{y}{x}$  for the values in the table. Explain why the relationship between number of tickets and total cost is not proportional.

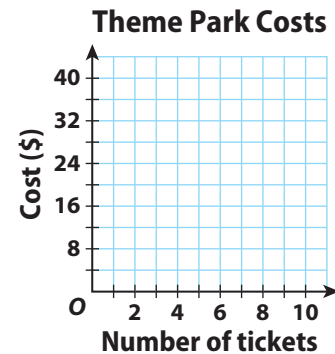
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### Reflect

2. **Analyze Relationships** Would it make sense to add more points to the graph from  $x = 0$  to  $x = 10$ ? Would it make sense to connect the points with a line? Explain.

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# Representing Linear Relationships Using Graphs

A **linear equation** is an equation whose solutions are ordered pairs that form a line when graphed on a coordinate plane. Linear equations can be written in the form  $y = mx + b$ . When  $b \neq 0$ , the relationship between  $x$  and  $y$  is *nonproportional*.



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## EXAMPLE 2



FL 8.F.1.3

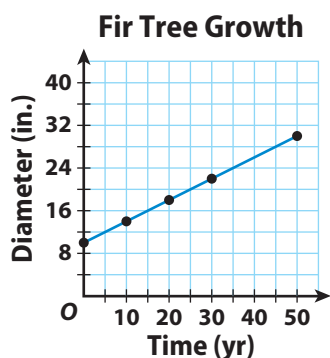
The diameter of a Douglas fir tree is currently 10 inches when measured at chest height. Over the next 50 years, the diameter is expected to increase by an average growth rate of  $\frac{2}{5}$  inch per year. The equation  $y = \frac{2}{5}x + 10$  gives  $y$ , the diameter of the tree in inches, after  $x$  years. Draw a graph of the equation. Describe the relationship.

**STEP 1** Make a table. Choose several values for  $x$  that make sense in context. To make calculations easier, choose multiples of 5.

$x$ (years)	0	10	20	30	50
$y$ (diameter in inches)	10	14	18	22	30

**STEP 2** Plot the ordered pairs from the table. Then draw a line connecting the points to represent all the possible solutions.

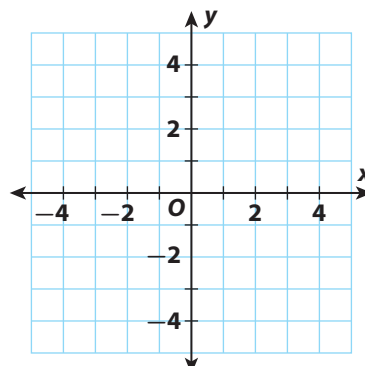
**STEP 3** The relationship is linear but nonproportional. The graph is a line but it does not go through the origin.



## YOUR TURN

3. Make a table and graph the solutions of the equation  $y = -2x + 1$ .

$x$	-1	0	1	2
$y$				



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My Notes

# Guided Practice

Make a table of values for each equation. (Example 1)

1.  $y = 2x + 5$

x	-2	-1	0	1	2
y					

2.  $y = \frac{3}{8}x - 5$

x	-8	0	8		
y					

Explain why each relationship is not proportional. (Explore Activity)

3.

x	0	2	4	6	8
y	3	7	11	15	19

First calculate  $\frac{y}{x}$  for the values in the table.

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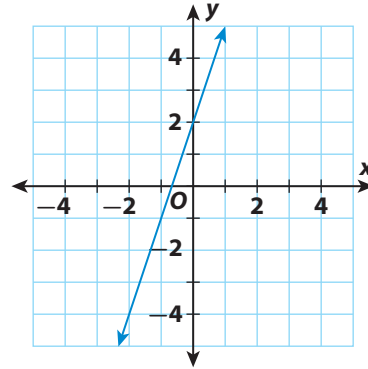


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4.




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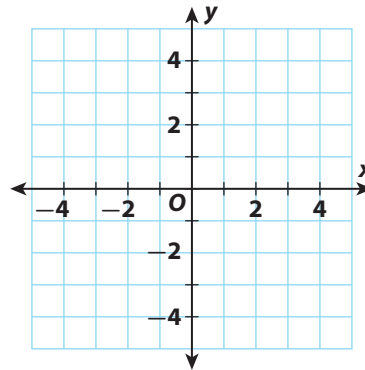


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Complete the table for the equation. Then use the table to graph the equation. (Example 2)

5.  $y = x - 1$

x	-2	-1	0	1	2
y					



## ESSENTIAL QUESTION CHECK-IN

6. How can you choose values for  $x$  when making a table of values representing a real world situation?

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# 4.1 Independent Practice

 **FL** 8.F.1.3



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**State whether the graph of each linear relationship is a solid line or a set of unconnected points. Explain your reasoning.**

- 7.** The relationship between the number of \$4 lunches you buy with a \$100 school lunch card and the money remaining on the card

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- 8.** The relationship between time and the distance remaining on a 3-mile walk for someone walking at a steady rate of 2 miles per hour

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- 9. Analyze Relationships** Simone paid \$12 for an initial year’s subscription to a magazine. The renewal rate is \$8 per year. This situation can be represented by the equation  $y = 8x + 12$ , where  $x$  represents the number of years the subscription is renewed and  $y$  represents the total cost.

- a.** Make a table of values for this situation.


- b.** Draw a graph to represent the situation. Include a title and axis labels.

- c.** Explain why this relationship is not proportional.

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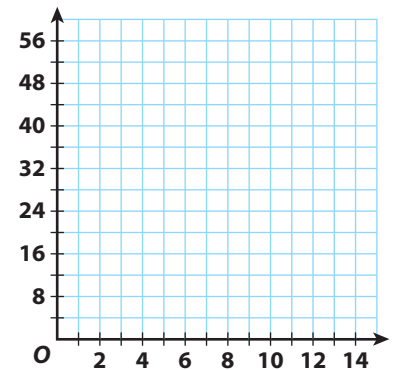
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- d.** Does it make sense to connect the points on the graph with a solid line? Explain.

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**10. Analyze Relationships** A proportional relationship is a linear relationship because the rate of change is constant (and equal to the constant of proportionality). What is required of a proportional relationship that is *not* required of a general linear relationship?

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**11. Communicate Mathematical Ideas** Explain how you can identify a linear non-proportional relationship from a table, a graph, and an equation.

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**H.O.T.** FOCUS ON HIGHER ORDER THINKING

**12. Critique Reasoning** George observes that for every increase of 1 in the value of  $x$ , there is an increase of 60 in the corresponding value of  $y$ . He claims that the relationship represented by the table is proportional. Critique George's reasoning.

$x$	1	2	3	4	5
$y$	90	150	210	270	330

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**13. Make a Conjecture** Two parallel lines are graphed on a coordinate plane. How many of the lines could represent proportional relationships? Explain.

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Work Area

# LESSON 4.2 Determining Slope and $y$ -intercept

 **FL** 8.EE.2.6

...; derive the equation  $y = mx$  for a line through the origin and the equation  $y = mx + b$  for a line intercepting the vertical axis at  $b$ . Also 8.F.2.4



## ESSENTIAL QUESTION

How can you determine the slope and the  $y$ -intercept of a line?

### EXPLORE ACTIVITY 1

 **FL** 8.EE.2.6

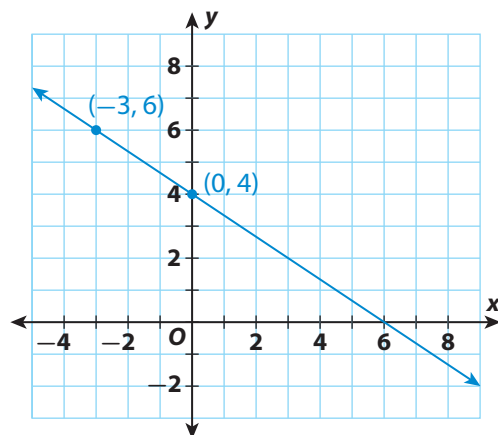
## Investigating Slope and $y$ -intercept

The graph of every nonvertical line crosses the  $y$ -axis. The  **$y$ -intercept** is the  $y$ -coordinate of the point where the graph intersects the  $y$ -axis. The  $x$ -coordinate of this point is always 0.

The graph represents the linear equation  $y = -\frac{2}{3}x + 4$ .

- STEP 1** Find the slope of the line using the points  $(0, 4)$  and  $(-3, 6)$ .

$$m = \frac{6 - \boxed{\phantom{00}}}{\boxed{\phantom{00}} - 0} = \frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}} = \boxed{\phantom{00}}$$



- STEP 2** The line also contains the point  $(6, 0)$ . What is the slope using  $(0, 4)$  and  $(6, 0)$ ? Using  $(-3, 6)$  and  $(6, 0)$ . What do you notice?

\_\_\_\_\_

- STEP 3** Compare your answers in Steps 1 and 2 with the equation of the graphed line.

\_\_\_\_\_  
\_\_\_\_\_

- STEP 3** Find the value of  $y$  when  $x = 0$  using the equation  $y = -\frac{2}{3}x + 4$ . Describe the point on the graph that corresponds to this solution.

\_\_\_\_\_

- STEP 5** Compare your answer in Step 3 with the equation of the line.

\_\_\_\_\_  
\_\_\_\_\_



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# Determining Rate of Change and Initial Value

The linear equation shown is written in the **slope-intercept form of an equation**. Its graph is a line with **slope  $m$**  and **y-intercept  $b$** .

$$y = mx + b$$

↑ slope                      ↓ y-intercept

A linear relationship has a constant rate of change. You can find the **rate of change  $m$**  and the **initial value  $b$**  for a linear situation from a table of values.

## EXAMPLE 1



FL 8.F.2.4

A phone salesperson is paid a minimum weekly salary and a commission for each phone sold, as shown in the table. Confirm that the relationship is linear and give the constant rate of change and the initial value.

**STEP 1** Confirm that the rate of change is constant.

$$\frac{\text{change in income}}{\text{change in phones sold}} = \frac{630 - 480}{20 - 10} = \frac{150}{10} = 15$$

$$\frac{\text{change in income}}{\text{change in phones sold}} = \frac{780 - 630}{30 - 20} = \frac{150}{10} = 15$$

$$\frac{\text{change in income}}{\text{change in phones sold}} = \frac{930 - 780}{40 - 30} = \frac{150}{10} = 15$$

Number of Phones Sold	Weekly Income (\$)
10	\$480
20	\$630
30	\$780
40	\$930

The rate of change is a constant, **15**.

The salesperson receives a \$15 commission for each phone sold.

**STEP 2** Find the initial value when the number of phones sold is 0.

Number of phones sold	0	10	20
Weekly income (\$)	330	480	630

-10   -10  
-150   -150

Work backward from  $x = 10$  to  $x = 0$  to find the initial value.

The initial value is \$330. The salesperson receives a salary of \$330 each week before commissions.

### Math Talk

Mathematical Practices

How do you use the rate of change to work backward to find the initial value?

## YOUR TURN

Find the slope and y-intercept of the line represented by each table.

1.

x	2	4	6	8
y	22	32	42	52

2.

x	1	2	3	4
y	8	15	22	29



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# Deriving the Slope-intercept Form of an Equation

In the following Explore Activity, you will derive the slope-intercept form of an equation.

**STEP 1** Let  $L$  be a line with slope  $m$  and  $y$ -intercept  $b$ . Circle the point that must be on the line. Justify your choice.

$(b, 0)$        $(0, b)$        $(0, m)$        $(m, 0)$

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**STEP 2** Recall that slope is the ratio of change in  $y$  to change in  $x$ . Complete the equation for the slope  $m$  of the line using the  $y$ -intercept  $(0, b)$  and another point  $(x, y)$  on the line.

$$m = \frac{y - \boxed{\phantom{0}}}{\boxed{\phantom{0}} - 0}$$

**STEP 3** In an equation of a line, we often want  $y$  by itself on one side of the equation. Solve the equation from Step 2 for  $y$ .

$$m = \frac{y - b}{x} \quad \text{Simplify the denominator.}$$

$$m \cdot \boxed{\phantom{x}} = \frac{y - b}{x} \cdot \boxed{\phantom{x}} \quad \text{Multiply both sides of the equation by } \underline{\hspace{2cm}}.$$

$$m \boxed{\phantom{x}} = y - b$$

$$mx + \boxed{\phantom{0}} = y - b + \boxed{\phantom{0}} \quad \text{Add } \underline{\hspace{2cm}} \text{ to both sides of the equation.}$$

$$mx + \boxed{\phantom{0}} = y$$

$$y = mx + \boxed{\phantom{0}} \quad \text{Write the equation with } y \text{ on the left side.}$$

## Reflect

3. **Critical Thinking** Write the equation of a line with slope  $m$  that passes through the origin. Explain your reasoning.

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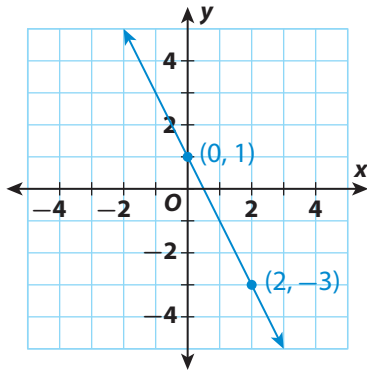


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# Guided Practice

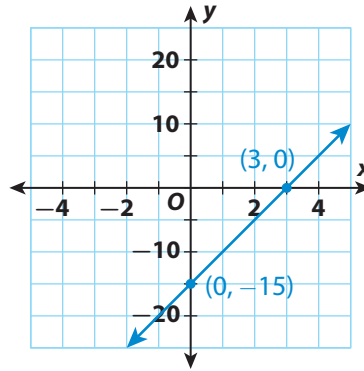
Find the slope and y-intercept of the line in each graph. (Explore Activity 1)

1.



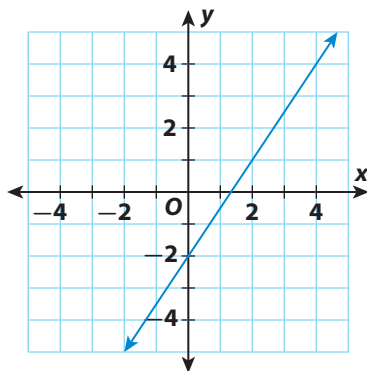
slope  $m =$  \_\_\_\_\_ y-intercept  $b =$  \_\_\_\_\_

2.



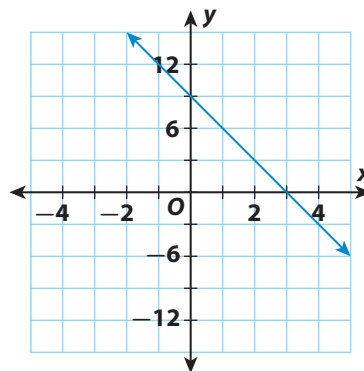
slope  $m =$  \_\_\_\_\_ y-intercept  $b =$  \_\_\_\_\_

3.



slope  $m =$  \_\_\_\_\_ y-intercept  $b =$  \_\_\_\_\_

4.



slope  $m =$  \_\_\_\_\_ y-intercept  $b =$  \_\_\_\_\_

Find the slope and y-intercept of the line represented by each table. (Example 1)

5.

$x$	0	2	4	6	8
$y$	1	7	13	19	25

slope  $m =$  \_\_\_\_\_ y-intercept  $b =$  \_\_\_\_\_

6.

$x$	0	5	10	15	20
$y$	140	120	100	80	60

slope  $m =$  \_\_\_\_\_ y-intercept  $b =$  \_\_\_\_\_

## ESSENTIAL QUESTION CHECK-IN

7. How can you determine the slope and the y-intercept of a line from a graph?

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# 4.2 Independent Practice



**FL** 8.EE.2.6, 8.F.2.4



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8. Some carpet cleaning costs are shown in the table. The relationship is linear. Find and interpret the rate of change and the initial value for this situation.

<b>Rooms cleaned</b>	1	2	3	4
<b>Cost (\$)</b>	125	175	225	275

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9. **Make Predictions** The total cost to pay for parking at a state park for the day and rent a paddleboat are shown.

- a. Find the cost to park for a day and the hourly rate to rent a paddleboat.


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- b. What will Lin pay if she rents a paddleboat for 3.5 hours and splits the total cost with a friend? Explain.

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Number of Hours	Cost (\$)
1	\$17
2	\$29
3	\$41
4	\$53

10. **Multi-Step** Raymond's parents will pay for him to take sailboard lessons during the summer. He can take half-hour group lessons or half-hour private lessons. The relationship between cost and number of lessons is linear.

<b>Lessons</b>	1	2	3	4
<b>Group (\$)</b>	55	85	115	145
<b>Private (\$)</b>	75	125	175	225

- a. Find the rate of change and the initial value for the group lessons.

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- b. Find the rate of change and the initial value for the private lessons.

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- c. Compare and contrast the rates of change and the initial values.

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**Vocabulary** Explain why each relationship is not linear.

11.

x	1	2	3	4
y	4.5	6.5	8.5	11.5

12.

x	3	5	7	9
y	140	126	110	92

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13. **Communicate Mathematical Ideas** Describe the procedure you performed to derive the slope-intercept form of a linear equation.

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**FOCUS ON HIGHER ORDER THINKING**

14. **Critique Reasoning** Your teacher asked your class to describe a real-world situation in which a y-intercept is 100 and the slope is 5. Your partner gave the following description: *My younger brother originally had 100 small building blocks, but he has lost 5 of them every month since.*

a. What mistake did your partner make?

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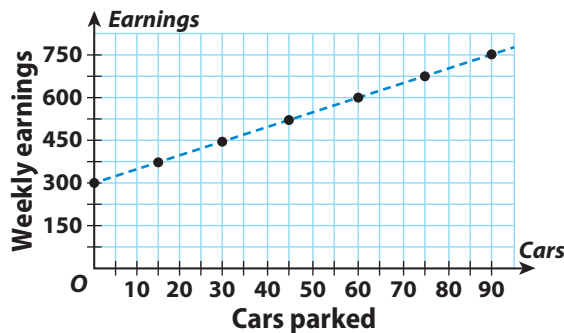
b. Describe a real-world situation that does match the situation.

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15. **Justify Reasoning** John has a job parking cars. He earns a fixed weekly salary of \$300 plus a fee of \$5 for each car he parks. His potential earnings for a week are shown in the graph. At what point does John begin to earn more from fees than his fixed salary? Justify your answer.




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Work Area



# LESSON 4.3

# Graphing Linear Nonproportional Relationships Using Slope and $y$ -intercept

 **FL** 8.F.2.4

... Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph. ... Also 8.F.1.3



## ESSENTIAL QUESTION

How can you graph a line using the slope and  $y$ -intercept?

## Using Slope-intercept Form to Graph a Line

Recall that  $y = mx + b$  is the slope-intercept form of the equation of a line. In this form, it is easy to see the slope  $m$  and the  $y$ -intercept  $b$ . So you can use this form to quickly graph a line by plotting the point  $(0, b)$  and using the slope to find a second point.

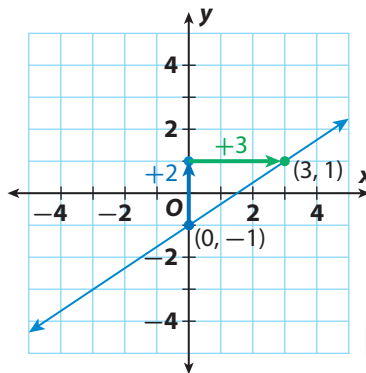


### EXAMPLE 1

 **FL** 8.F.1.3

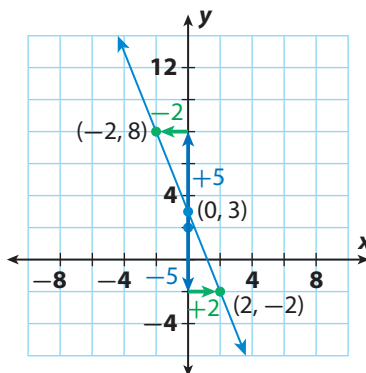
**A** Graph  $y = \frac{2}{3}x - 1$ .

- STEP 1** The  $y$ -intercept is  $b = -1$ . Plot the point that contains the  $y$ -intercept:  $(0, -1)$ .
- STEP 2** The slope is  $m = \frac{2}{3}$ . Use the slope to find a second point. From  $(0, -1)$ , count *up 2* and *right 3*. The new point is  $(3, 1)$ .
- STEP 3** Draw a line through the points.

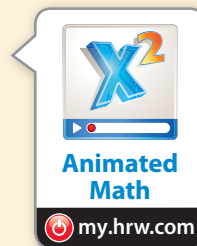


**B** Graph  $y = -\frac{5}{2}x + 3$ .

- STEP 1** The  $y$ -intercept is  $b = 3$ . Plot the point that contains the  $y$ -intercept:  $(0, 3)$ .
- STEP 2** The slope is  $m = -\frac{5}{2}$ . Use the slope to find a second point. From  $(0, 3)$ , count *down 5* and *right 2*, or *up 5* and *left 2*. The new point is  $(2, -2)$  or  $(-2, 8)$ .
- STEP 3** Draw a line through the points.



Note that the line passes through all three points:  $(-2, 8)$ ,  $(0, 3)$ , and  $(2, -2)$ .



**Math Talk**  
Mathematical Practices

Is a line with a positive slope always steeper than a line with a negative slope? Explain.


## Reflect

- Draw Conclusions** How can you use the slope of a line to predict the way the line will be slanted? Explain.

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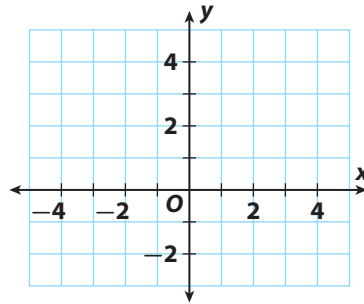


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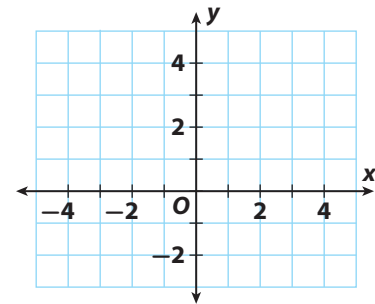
## YOUR TURN

Graph each equation.

2.  $y = \frac{1}{2}x + 1$



3.  $y = -3x + 4$




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## Analyzing a Graph

Many real-world situations can be represented by linear relationships. You can use graphs of linear relationships to visualize situations and solve problems.

### EXAMPLE 2



FL 8.F.2.4

Ken has a weekly goal of burning 2400 calories by taking brisk walks. The equation  $y = -300x + 2400$  represents the number of calories  $y$  Ken has left to burn after  $x$  hours of walking which burns 300 calories per hour.

- A** Graph the equation  $y = -300x + 2400$ .

**STEP 1** Write the slope as a fraction.

$$m = \frac{-300}{1} = \frac{-600}{2} = \frac{-900}{3}$$

Using the slope as  $-\frac{900}{3}$  helps in drawing a more accurate graph.

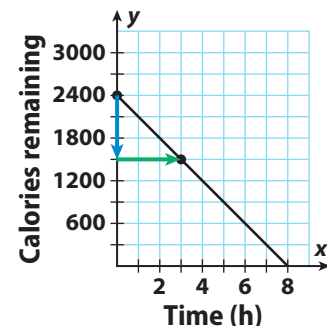
**STEP 2** Plot the point for the y-intercept: (0, 2400).

**STEP 3** Use the slope to locate a second point.

From (0, 2400), count *down* 900 and *right* 3.

The new point is (3, 1500).

**STEP 4** Draw a line through the two points.



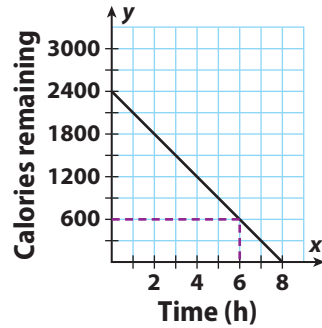
- B** After how many hours of walking will Ken have 600 calories left to burn? After how many hours will he reach his weekly goal?

**STEP 1** Locate 600 calories on the y-axis. Read across and down to the x-axis.

Ken will have 600 calories left to burn after 6 hours.

**STEP 2** Ken will reach his weekly goal when the number of calories left to burn is 0. Because every point on the x-axis has a y-value of 0, find the point where the line crosses the x-axis.

Ken will reach his goal after 8 hours of brisk walking.



## YOUR TURN

**What If?** Ken decides to modify his exercise plans from Example 2 by slowing the speed at which he walks. The equation for the modified plan is  $y = -200x + 2400$ .

- Graph the equation.
- How does the graph of the new equation compare with the graph in Example 2?

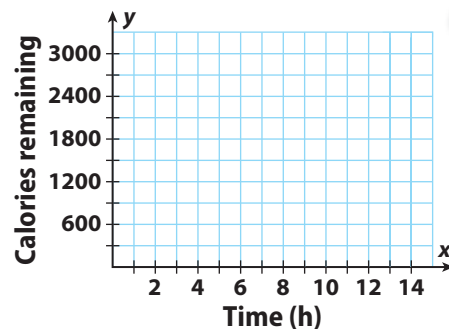
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- Will Ken have to exercise more or less to meet his goal? Explain.

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- Suppose that Ken decides that instead of walking, he will jog, and that jogging burns 600 calories per hour. How do you think that this would change the graph?

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## Math Talk

Mathematical Practices



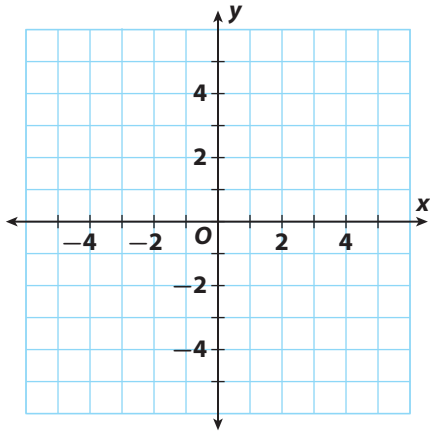
What do the slope and the y-intercept of the line represent in this situation?

## Guided Practice

Graph each equation using the slope and the y-intercept. (Example 1)

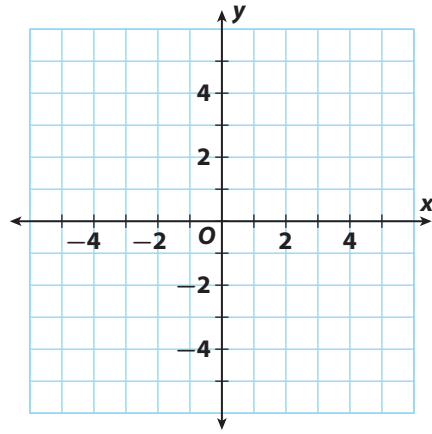
1.  $y = \frac{1}{2}x - 3$

slope = \_\_\_\_\_ y-intercept = \_\_\_\_\_



2.  $y = -3x + 2$

slope = \_\_\_\_\_ y-intercept = \_\_\_\_\_



3. A friend gives you two baseball cards for your birthday. Afterward, you begin collecting them. You buy the same number of cards once each week. The equation  $y = 4x + 2$  describes the number of cards,  $y$ , you have after  $x$  weeks. (Example 2)

- a. Find and interpret the slope and the y-intercept of the line that represents this situation. Graph  $y = 4x + 2$ . Include axis labels.

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- b. Discuss which points on the line do not make sense in this situation. Then plot three more points on the line that do make sense.

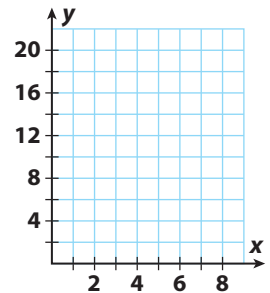
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### ESSENTIAL QUESTION CHECK-IN

4. Why might someone choose to use the y-intercept and the slope to graph a line?

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# 4.3 Independent Practice

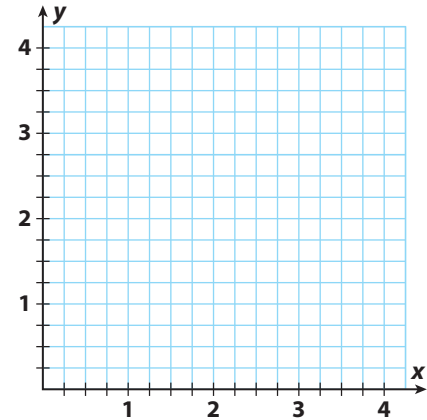


**FL** 8.F.1.3, 8.F.2.4



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**5. Science** A spring stretches in relation to the weight hanging from it according to the equation  $y = 0.75x + 0.25$  where  $x$  is the weight in pounds and  $y$  is the length of the spring in inches.



- a. Graph the equation. Include axis labels.
- b. Interpret the slope and the y-intercept of the line.

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c. How long will the spring be if a 2-pound weight is hung on it? Will the length double if you double the weight? Explain

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**Look for a Pattern** Identify the coordinates of four points on the line with each given slope and y-intercept.

6. slope = 5, y-intercept = -1

---

7. slope = -1, y-intercept = 8

---

8. slope = 0.2, y-intercept = 0.3

---

9. slope = 1.5, y-intercept = -3

---

10. slope =  $-\frac{1}{2}$ , y-intercept = 4

---

11. slope =  $\frac{2}{3}$ , y-intercept = -5

---

12. A music school charges a registration fee in addition to a fee per lesson. Music lessons last 0.5 hour. The equation  $y = 40x + 30$  represents the total cost  $y$  of  $x$  lessons. Find and interpret the slope and y-intercept of the line that represents this situation. Then find four points on the line.

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13. A public pool charges a membership fee and a fee for each visit. The equation  $y = 3x + 50$  represents the cost  $y$  for  $x$  visits.

a. After locating the  $y$ -intercept on the coordinate plane shown, can you move up three gridlines and right one gridline to find a second point? Explain.

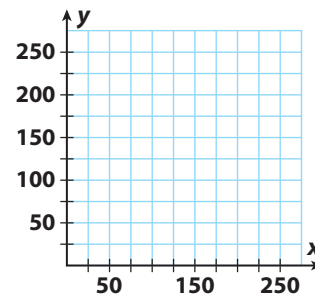
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b. Graph the equation  $y = 3x + 50$ . Include axis labels. Then interpret the slope and  $y$ -intercept.

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c. How many visits to the pool can a member get for \$200?

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**H.O.T.** FOCUS ON HIGHER ORDER THINKING

14. **Explain the Error** A student says that the slope of the line for the equation  $y = 20 - 15x$  is 20 and the  $y$ -intercept is 15. Find and correct the error.

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15. **Critical Thinking** Suppose you know the slope of a linear relationship and a point that its graph passes through. Can you graph the line even if the point provided does *not* represent the  $y$ -intercept? Explain.

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16. **Make a Conjecture** Graph the lines  $y = 3x$ ,  $y = 3x - 3$ , and  $y = 3x + 3$ . What do you notice about the lines? Make a conjecture based on your observation.

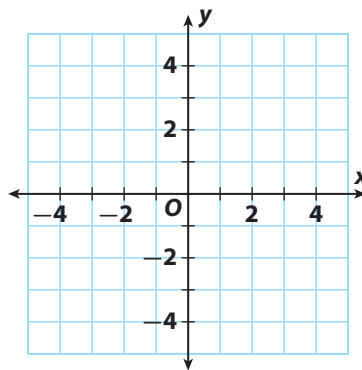
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Work Area

# LESSON 4.4 Proportional and Nonproportional Situations

 **FL** 8.F.1.2

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Also 8.F.1.3, 8.F.2.4



## ESSENTIAL QUESTION

How can you distinguish between proportional and nonproportional situations?

## Distinguish Between Proportional and Nonproportional Situations Using a Graph

If a relationship is nonlinear, it is nonproportional. If it is linear, it may be either proportional or nonproportional. When the graph of the linear relationship contains the origin, the relationship is proportional.



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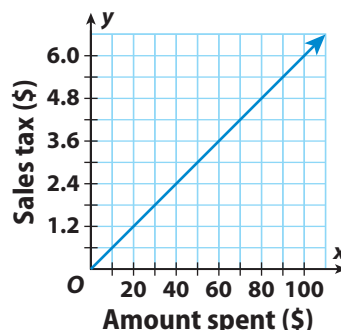
### EXAMPLE 1



 **FL** 8.F.1.3

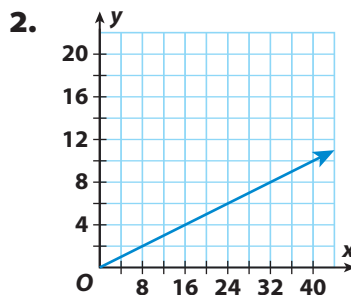
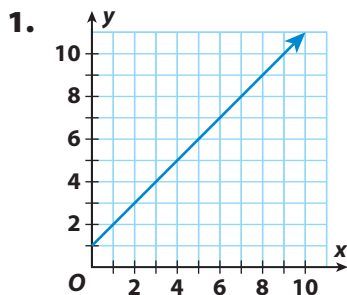
The graph shows the sales tax charged based on the amount spent at a video game store in a particular city. Does the graph show a linear relationship? Is the relationship proportional or nonproportional?

The graph shows a linear proportional relationship because it is a line that contains the origin.



### YOUR TURN

Determine if each of the following graphs represents a proportional or nonproportional relationship.



\_\_\_\_\_

\_\_\_\_\_

### Math Talk



Mathematical Practices

What do the slope and the y-intercept of the graph represent in this situation?



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# Distinguish Between Proportional and Nonproportional Situations Using an Equation

If an equation is not a linear equation, it represents a nonproportional relationship. A linear equation of the form  $y = mx + b$  may represent either a proportional ( $b = 0$ ) or nonproportional ( $b \neq 0$ ) relationship.

## EXAMPLE 2



FL

8.F.2.4

The number of years since Keith graduated from middle school can be represented by the equation  $y = a - 14$ , where  $y$  is the number of years and  $a$  is his age. Is the relationship between the number of years since Keith graduated and his age proportional or nonproportional?

$$y = a - 14$$

The equation is in the form  $y = mx + b$ , with  $a$  being used as the variable instead of  $x$ . The value of  $m$  is 1, and the value of  $b$  is  $-14$ . Since  $b$  is not 0, the relationship between the number of years since Keith graduated and his age is nonproportional.

### Reflect

3. **Communicate Mathematical Ideas** In a proportional relationship, the ratio  $\frac{y}{x}$  is constant. Show that this ratio is not constant for the equation  $y = a - 14$ .

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4. **What If?** Suppose another equation represents Keith's age in months  $y$  given his age in years  $a$ . Is this relationship proportional? Explain.

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### YOUR TURN

Determine if each of the following equations represents a proportional or nonproportional relationship.

5.  $d = 65t$

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6.  $p = 0.1s + 2000$

---

7.  $n = 450 - 3p$

---

8.  $36 = 12d$

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# Distinguish Between Proportional and Nonproportional Situations Using a Table

If there is not a constant rate of change in the data displayed in a table, then the table represents a nonlinear nonproportional relationship.

A linear relationship represented by a table is a proportional relationship when the quotient of each pair of numbers is constant. Otherwise, the linear relationship is nonproportional.

## EXAMPLE 3



FL 8.F.2.4

The values in the table represent the numbers of U.S. dollars three tourists traded for Mexican pesos. The relationship is linear. Is the relationship proportional or nonproportional?

U.S. Dollars Traded	Mexican Pesos Received
130	1,690
255	3,315
505	6,565



$$\frac{1,690}{130} = \frac{169}{13} = 13$$

$$\frac{3,315}{255} = \frac{221}{17} = 13$$

$$\frac{6,565}{505} = \frac{1313}{101} = 13$$

Simplify the ratios to compare the pesos received to the dollars traded.

The ratio of pesos received to dollars traded is constant at 13 Mexican pesos per U.S. dollar. This is a proportional relationship.

## YOUR TURN

Determine if the linear relationship represented by each table is a proportional or nonproportional relationship.

9.

x	y
2	30
8	90
14	150

10.

x	y
5	1
40	8
65	13



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## Math Talk

Mathematical Practices

How could you confirm that the values in the table have a linear relationship?



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# Comparing Proportional and Nonproportional Situations

You can use what you have learned about proportional and nonproportional relationships to compare similar real-world situations that are given using different representations.

## EXAMPLE 4



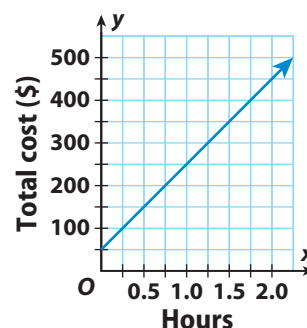
FL 8.F.1.2

- A** A laser tag league has the choice of two arenas for a tournament. In both cases,  $x$  is the number of hours and  $y$  is the total charge. Compare and contrast these two situations.

**Arena A**

$$y = 225x$$

**Arena B**



- **Arena A's** equation has the form  $y = mx + b$ , where  $b = 0$ . So, Arena A's charges are a proportional relationship. The hourly rate, \$225, is greater than Arena B's, but there is no additional fee.
  - **Arena B's** graph is a line that does not include the origin. So, Arena B's charges are a nonproportional relationship. Arena B has a \$50 initial fee but its hourly rate, \$200, is lower.
- B** Jessika is remodeling and has the choice of two painters. In both cases,  $x$  is the number of hours and  $y$  is the total charge. Compare and contrast these two situations.

**Painter A**

$$y = \$45x$$

**Painter B**

$x$	0	1	2	3
$y$	20	55	90	125

**Painter A's** equation has the form  $y = mx + b$ , where  $b = 0$ . So, Painter A's charges are proportional. The hourly rate, \$45, is greater than Painter B's, but there is no additional fee.

**Painter B's** table is a nonproportional relationship because the ratio of  $y$  to  $x$  is not constant. Because the table contains the ordered pair  $(0, 20)$ , Painter B charges an initial fee of \$20, but the hourly rate, \$35, is less than Painter A's.

### Math Talk

#### Mathematical Practices

How might graphing the equation for Arena A help you to compare the situations?

## YOUR TURN

11. Compare and contrast the following two situations.

Test-Prep Center A	Test-Prep Center B
The cost for Test-Prep Center A is given by $c = 20h$ , where $c$ is the cost in dollars and $h$ is the number of hours you attend.	Test-Prep Center B charges \$25 per hour to attend, but you have a \$100 coupon that you can use to reduce the cost.

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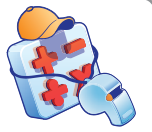
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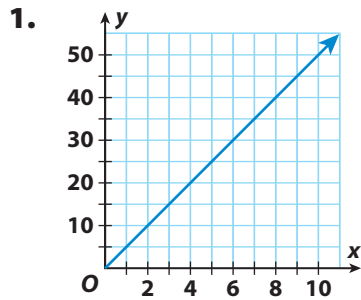
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## Guided Practice

Determine if each relationship is a proportional or nonproportional situation. Explain your reasoning. (Example 1, Example 2, Example 4)

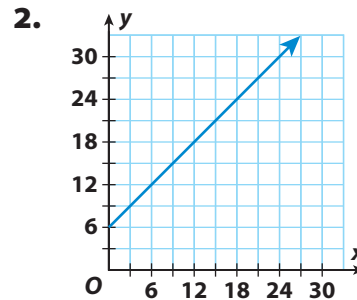


Look at the origin.

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3.  $q = 2p + \frac{1}{2}$

Compare the equation with  $y = mx + b$ .

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4.  $v = \frac{1}{10}u$

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The tables represent linear relationships. Determine if each relationship is a proportional or nonproportional situation. (Example 3, Example 4)

5.

$x$	$y$
3	12
9	36
21	84

6.

$x$	$y$
22	4
46	8
58	10

Find the quotient of  $y$  and  $x$ .

_____	_____
_____	_____
_____	_____

7. The values in the table represent the numbers of households that watched three TV shows and the ratings of the shows. The relationship is linear. Describe the relationship in other ways. (Example 4)

Number of Households that Watched TV Show	TV Show Rating
15,000,000	12
20,000,000	16
25,000,000	20

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

 **ESSENTIAL QUESTION CHECK-IN**

8. How are using graphs, equations, and tables similar when distinguishing between proportional and nonproportional linear relationships?

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# 4.4 Independent Practice



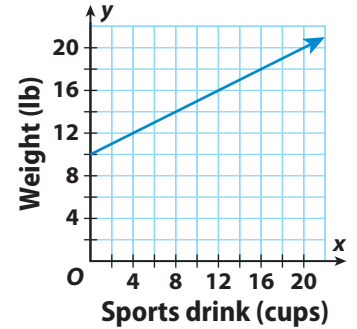
**FL** 8.F.1.2, 8.F.1.3, 8.F.2.4

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9. The graph shows the weight of a cross-country team's beverage cooler based on how much sports drink it contains.



a. Is the relationship proportional or nonproportional? Explain.

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b. Identify and interpret the slope and the y-intercept.

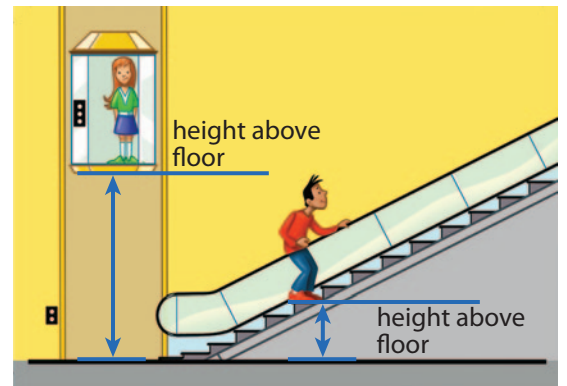
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In 10–11, tell if the relationship between a rider's height above the first floor and the time since the rider stepped on the elevator or escalator is proportional or nonproportional. Explain your reasoning.

10. The elevator paused for 10 seconds after you stepped on before beginning to rise at a constant rate of 8 feet per second.




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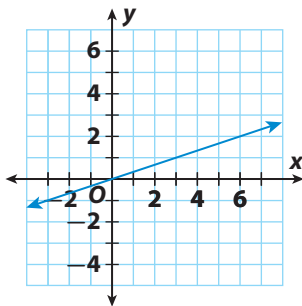
11. Your height,  $h$ , in feet above the first floor on the escalator is given by  $h = 0.75t$ , where  $t$  is the time in seconds.

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12. **Analyze Relationships** Compare and contrast the two graphs.

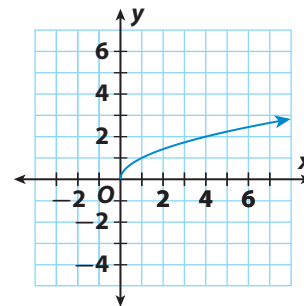
**Graph A**

$$y = \frac{1}{3}x$$



**Graph B**

$$y = \sqrt{x}$$




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13. **Represent Real-World Problems** Describe a real-world situation where the relationship is linear and nonproportional.

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**FOCUS ON HIGHER ORDER THINKING**

14. **Mathematical Reasoning** Suppose you know the slope of a linear relationship and one of the points that its graph passes through. How can you determine if the relationship is proportional or nonproportional?

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15. **Multiple Representations** An entrant at a science fair has included information about temperature conversion in various forms, as shown. The variables  $F$ ,  $C$ , and  $K$  represent temperatures in degrees Fahrenheit, degrees Celsius, and Kelvin, respectively.

Equation A $F = \frac{9}{5}C + 32$	Table C	
	Degrees Celsius	kelvins
Equation B $K = C + 273.15$	8	281.15
	15	288.15
	36	309.15

- a. Is the relationship between kelvins and degrees Celsius proportional? Justify your answer in two different ways.

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- b. Is the relationship between degrees Celsius and degrees Fahrenheit proportional? Why or why not?

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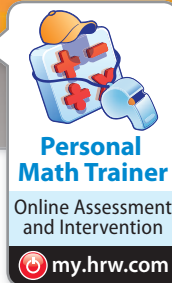
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Work Area

# Ready to Go On?



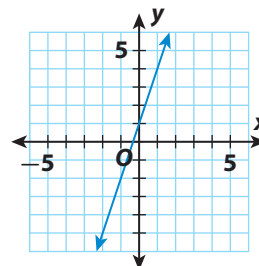
## 4.1 Representing Linear Nonproportional Relationships

1. Complete the table using the equation  $y = 3x + 2$ .

x	-1	0	1	2	3
y					

## 4.2 Determining Slope and y-intercept

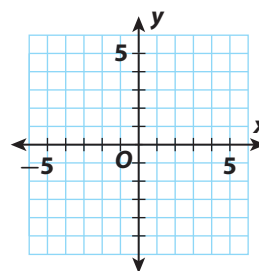
2. Find the slope and y-intercept of the line in the graph.



\_\_\_\_\_

## 4.3 Graphing Linear Nonproportional Relationships

3. Graph the equation  $y = 2x - 3$  using slope and y-intercept.



## 4.4 Proportional and Nonproportional Situations

4. Does the table represent a proportional or a nonproportional linear relationship?

x	1	2	3	4	5
y	4	8	12	16	20

\_\_\_\_\_

5. Does the graph in Exercise 2 represent a proportional or a nonproportional linear relationship?

\_\_\_\_\_

6. Does the graph in Exercise 3 represent a proportional or a nonproportional relationship?

\_\_\_\_\_



### ESSENTIAL QUESTION

7. How can you identify a linear nonproportional relationship from a table, a graph, and an equation?

\_\_\_\_\_  
\_\_\_\_\_



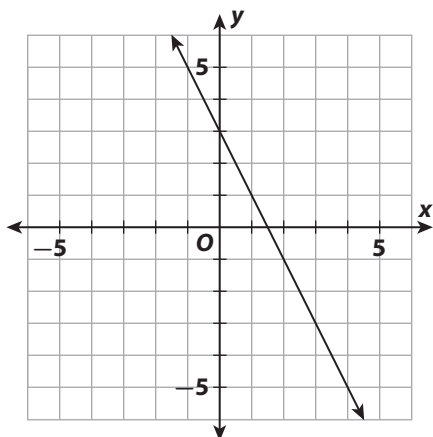
## Selected Response

1. The table below represents which equation?

<b>x</b>	-1	0	1	2
<b>y</b>	-10	-6	-2	2

- (A)  $y = -x - 10$       (C)  $y = 4x - 6$   
 (B)  $y = -6x$       (D)  $y = -4x + 2$

2. The graph of which equation is shown below?



- (A)  $y = -2x + 3$       (C)  $y = 2x + 3$   
 (B)  $y = -2x + 1.5$       (D)  $y = 2x + 1.5$

3. The table below represents a linear relationship.

<b>x</b>	2	3	4	5
<b>y</b>	4	7	10	13

What is the y-intercept?

- (A) -4      (C) 2  
 (B) -2      (D) 3

4. Which equation represents a nonproportional relationship?

- (A)  $y = 3x + 0$       (C)  $y = 3x + 5$   
 (B)  $y = -3x$       (D)  $y = \frac{1}{3}x$

5. The table shows a proportional relationship. What is the missing y-value?

<b>x</b>	4	10	12
<b>y</b>	6	15	?

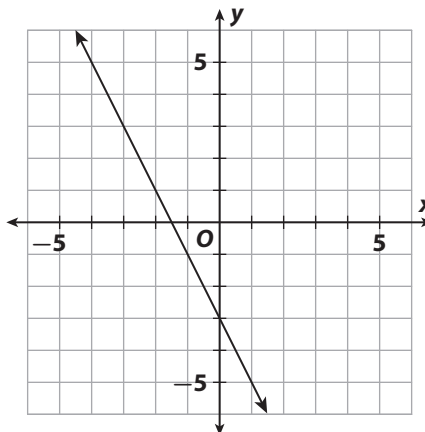
- (A) 16      (C) 18  
 (B) 20      (D) 24

6. What is 0.00000598 written in scientific notation?

- (A)  $5.98 \times 10^{-6}$       (C)  $59.8 \times 10^{-6}$   
 (B)  $5.98 \times 10^{-5}$       (D)  $59.8 \times 10^{-7}$

## Mini-Task

7. The graph shows a linear relationship.



a. Is the relationship proportional or nonproportional?

\_\_\_\_\_

b. What is the slope of the line?

\_\_\_\_\_

c. What is the y-intercept of the line?

\_\_\_\_\_

d. What is the equation of the line?

\_\_\_\_\_